TECHNICAL NOTE

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Fractal Surfaces as Models of Physical Matches

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ABSTRACT: Computer-generated fractal surfaces may be used as models of evidence physical matches. The complexity of the model surface, and by extension, the degree of uniqueness of the surface, may be expressed in terms of the time required for the calculation of the surface parameters.

KEYWORDS: forensic science, fractures (materials), matching, fractal

A physical match between two fractured surfaces is routinely viewed, and quite correctly, as definitive proof that the surfaces had at one time been joined. Although it is ordinarily a straightforward matter to show the "fit" between the complementary fractured surfaces, there has been little systematic effort within the forensic science community to establish any fundamental or theoretical basis for the uniqueness of a physical match. Walls [*1*] took the position that a fracture propagation (in one plane) is a constrained meandering, and invoked the statistical concept of "random walks." Walls' model is based on possible "inflection points" at undefined incremental distances, and results in a probability that the fracture has followed a particular course in one plane.

The present work takes a different approach, in which model surfaces are generated which are reminiscent of actual three-dimensional fracture surfaces. The propagation of the model surfaces is expressed, not in terms of probability, but in terms of the time required to calculate the parameters of the surface.

The model surfaces employed for this purpose are computer-generated fractal surfaces. [The neologism "fractal" and the word "fracture" share the same root (*fractus*, adj., from the Latin *frangere*, v., "to break").] Fractals are curves or surfaces of "fractional dimension" in which the topological dimension strictly exceeds the Hausdorf-Besikovich dimension; they have been extensively reviewed by Mandelbrot [*2,3*]. Although a full explanation is unnecessary here, a fractal may be viewed as a dimensionally discordant figure which is defined by a real number. This real number, on both intuitive and formal grounds, deserves to be called the dimension of the figure. We frequently deal with figures in two or three dimensions; fractals, on the other hand, need not have dimensions that are integers. For example, a fractal may be described by dimension 1.2244 or 2.2816. Fractal geometry would

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seem applicable to the description of fracture surfaces, as well as toolmarks and other striated surfaces, because of the unique ability of fractals to describe natural forms. Nature exhibits not just a higher degree of complexity over conventional Euclidean geometry, but an altogether different level of complexity which fractals may appropriately address.

A number of obscure computer programs have been written for the construction of fractal surfaces, some of the best of which have been developed by Lucasfilm; one accessable program, however, is that of van de Panne [*4*]. This program, in the Apple II version, was used in the present work. (The van Panne program, as originally published, is capable of plotting fractal surfaces along three axes, resulting in a figure with twice as many surfaces as those depicted here and comprised entirely of triangles. The resolution in the "Level 6" surface presented below approaches pixel size, however; the van de Panne program was therefore modified to plot a "net figure" comprised of quadrilaterals, instead of triangles, in order that the figures would not be excessively cluttered.)

Figure 1 illustrates six computer generated fractal surfaces and the surface from which they are derived. Three points constitute a plane, and so the triangle seen in Fig. 1*a* may be construed as a surface in object space. A plane surface such as this could not reasonably enter into a physical match because it is featureless. But we may now rearrange and build onto this surface, imparting features and "personality" to the surface which will ultimately result in a realistic approximation of an actual fracture surface.

The midpoints of each of the sides of a triangle may be randomly raised or lowered relative to a "sea level," the amount of raising or lowering being proportional to the length of the side of the triangle. The midpoint of one side of the triangle may then be joined to the midpoints of the other two sides to form a "net figure" as depicted in Fig. 1*b*. This represents a fractal surface of Level 1. The surface is now partitioned into three separate surfaces, a quadrilateral and two triangles. Although this surface now has some minimal degree of individuality, it is scarcely realistic as a model of a fracture surface.

Repeating the process again of randomly raising or lowering the midpoints of the sides of a triangle and joining the midpoint of one side with the midpoints of the other two sides, we may derive a fractal surface of Level 2, comprised of ten plane surfaces as depicted in Fig. 1*c*. Continuing this through six levels (the limit of memory of a 64K RAM Apple II+), we obtain surfaces such as those depicted in Fig. 1*d* through *g*. Although Fig. 1*b* through *e* are too naive to serve as realistic models of a fracture surface, Fig. 1*g* is of such complexity as to be reminiscent of an actual fracture surface. Figure 1*f* might arguably qualify also.

We are now in a position to make a quantitative statement as to the complexity, and therefore of the individuality, of the surface. We are able to do so because we may document the calculations required by the computer to construct the surface.

The calculation and plotting time for the Level 1 fractal surface (Fig. 1*b*) with the Apple II+ was determined to be 15 s; for the Level 2 surface the time was 28 s; for the Level 3 fractal, 67 s; for the Level 4 surface, 205 s; for the Level 5 surface, 717 s; and for the Level 6 surface (Fig. 1*g*) the time was 2740 s (or 45 min, 40 s).

Typical crack propagation velocity in solids is on the order of 2400 m/s [*5*]. A fracture surface of 1 $in.^2$ (6.45 cm^2) could therefore be generated in approximately 10 μs. If a computer with a microprocessor capable of 500 000 operations per second (for example, the Apple II+) requires 2740 s to produce a reasonable approximation of a real fracture surface, it is working approximately 2½ billion times slower than the processes at work in the generation of actual fractures. Stated differently, the Apple computer would have to work 2½ billion times faster if it had to generate a significant fracture surface in real time. There is no doubt but that other, more sophisticated, computers would require less time to construct the surfaces, but they would also require less time to construct the simple surfaces, for example, Fig. 1*b* through *e*; it is the *relative* time that is at issue here.

The significance of these measures of the relative time required to calculate surface parameters is that if we accept Walls' conceptual view of a fracture, that is, that the fracture "decides" which course to follow in an iterative fashion, then in actual fractures these "deci-

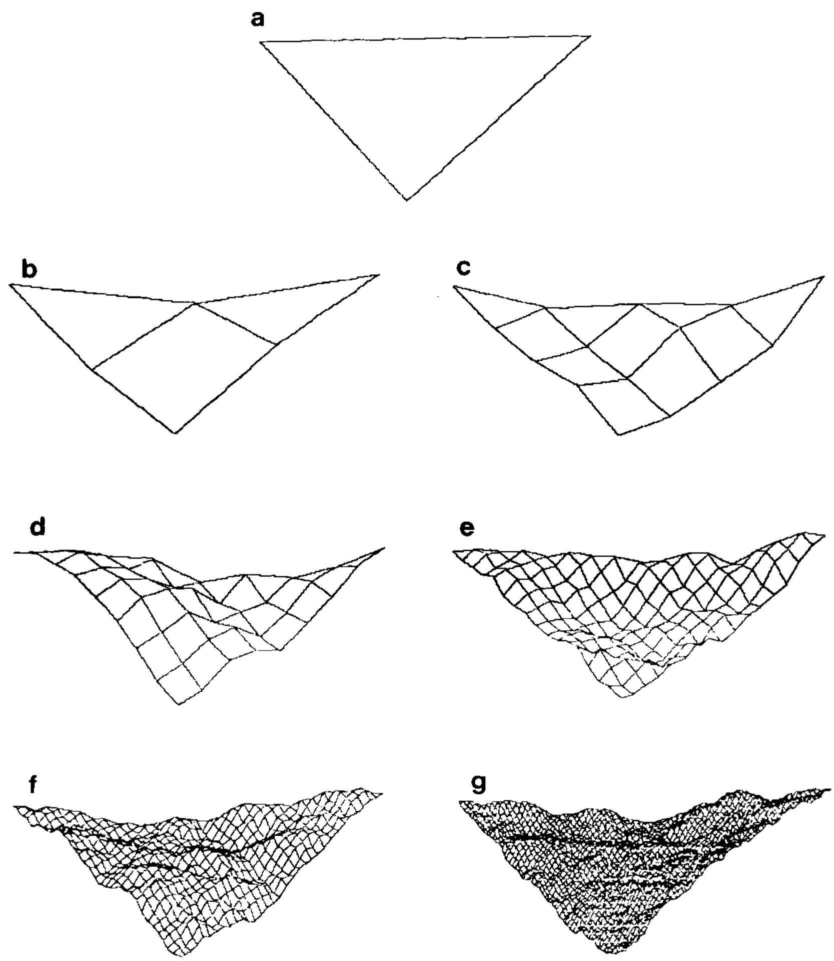

FIG. 1—*Computer-generated fractal surfaces:* (a) *represents a plane surface, and* (b) *through* (g) *represents fractal surfaces of Levels 1 through 6, respectively. The time required for the computation of surface parameters using the modification of the van Panne program* [2] *is as follows:* (b) *Level 1—15 s;* (c) *Level 2—28 s;* (d) *Level 3—67 s;* (e) *Level 4— 205 s;* (f) *Level 5—717 s;* (g) *Level 6—2740 s.*

sions" are made in an incredibly short period of time and are expressed in a surface of such complexity that a computer would have to work for billions of times longer to mimic the actual fracture. In the view of the present author, the framing of the complexity of a fractal or fracture surface in terms of the time required for its generation represents a means for the conceptualization of the intricacy and uniqueness of an actual fracture. Fracture surfaces of even moderate complexity could not credibly be subject to adventitious replication; there is no physical basis to explain how extrinsic factors capable of influencing the nature of the fracture surface could operate within such a short period of time.

References

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